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Therefore, when

$$x = \frac{a}{\sqrt{3}}, \qquad y = \frac{b}{\sqrt{3}}, \qquad \Delta \equiv \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - \left(\frac{\partial^2 F}{\partial y \partial x}\right)^2 = 4;$$

and since

$$\frac{\partial^2 F}{\partial x^2} < 0, \qquad \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = 0,$$

we have all the conditions for a maximum value of $F(x, y)$ fulfilled at $(a/\sqrt{3}, b/\sqrt{3})$.

Hence, finally, substituting in (1), we have the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid equal to $8abc/(3\sqrt{3})$.

Also solved by L. E. Lunn, L. E. Mensenkamp, O. S. Adams, C. E. Githens, J. L. Riley, Paul Capron, and H. C. Feemster.

441 (Calculus). Proposed by J. L. RILEY, Stephenville, Texas.

Find the minimum value of

$$\int \left\{ \left(\frac{dy}{dx}\right)^2 \sin x + (y + x - \sin x)^2/\sin x \right\} dx.$$

Solution by Elijah Swift, University of Vermont.

This problem is indefinite and no solution is possible, until the conditions that the end points must satisfy are stated. In fact, if we take $y = mx$ and integrate from $\pi + \epsilon$ to $2\pi - \epsilon$, we can make the integral as small as desired by decreasing ϵ.

It is not difficult to find the equation of the extremals. Euler's differential equation is $d(F_{y'})/dx - F_y = 0$. This becomes for our problem $y'' \sin^2 x + y' \sin x \cos x - y = x - \sin x$. A solution of the equation where we make the right-hand member zero is

$$y = C_1 \tan (x/2) + C_2 \operatorname{ctn} (x/2),$$

obtained by taking $2y'$ as an obvious integrating factor. Completing the solution by any one of several methods, there results

$$y = C_1 \tan (x/2) + C_2 \operatorname{ctn} (x/2) - x + \operatorname{ctn} (x/2) \log \sec^2 (x/2).$$

For any further investigation of the problem, however, a knowledge of the boundary conditions is necessary.

Also solved by Alexander Dillingham.

349 (Mechanics). Proposed by S. A. COREY, Albia, Iowa.

A 9-pound weight is attached to a string which passes over a smooth fixed pulley. The other end of the string is fastened to and supports a smooth pulley P_1 of weight 1 pound, over which passes a second string to one end of which is attached a 3-pound weight, and the other end of which is attached to and supports another smooth pulley P_2 of weight 1 pound. Over the pulley P_2 passes a third string supporting weights, 2 pounds and $3\frac{1}{3}$ pounds.

If the system is acted on by gravity alone show that the accelerations of the 9-pound weight, $3\frac{1}{3}$-pound weight, and pulley P_2 are 0, $\frac{1}{8}g$, and $\frac{1}{3}g$, respectively.

Determine the motion of the weights when pulleys are not smooth, that is, when friction is present.

Solution by the Proposer.

Let x = distance of 9-pound weight from center of fixed pulley, y = distance of center of P_2 from center of P_1, and z = distance of $3\frac{1}{3}$-pound weight from center of P_2.

Then will $\dot{x}$ = velocity of 9-pound weight, $-\dot{x}$ = velocity of P_1, $\dot{y} - \dot{x}$ = velocity of P_2 $-\dot{y} - \dot{x}$ = velocity of 3-pound weight, $\dot{z} + \dot{y} - \dot{x}$ = velocity of $3\frac{1}{3}$-pound weight, and $-\dot{z} + \dot{y} - \dot{x}$ = velocity of 2-pound weight.

If T = kinetic energy of entire moving system, and if m represent mass of 1-pound weight, we have

$$T = \frac{m}{2}\Big[9\dot{x}^2 + \dot{x}^2 + (\dot{y}^2 - 2\dot{x}\dot{y} + \dot{x}^2) + 3(\dot{y}^2 + 2\dot{y}\dot{x} + \dot{x}^2) + \tfrac{10}{3}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 - 2\dot{x}\dot{y}$$

$$+ 2\dot{z}\dot{y} - 2\dot{z}\dot{x}) + 2(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2\dot{x}\dot{z} - 2\dot{y}\dot{z} - 2\dot{x}\dot{y})\Big]$$

$$= \frac{m}{3}(29\dot{x}^2 + 14\dot{y}^2 + 8\dot{z}^2 - 10\dot{x}\dot{y} + 4\dot{y}\dot{z} - 4\dot{x}\dot{z});$$

whence,

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = \frac{2m}{3}(29\ddot{x} - 5\ddot{y} - 2\ddot{z}) = (9 - 1 - 3 - 1 - 2 - \tfrac{10}{3})mg = -\tfrac{4}{3}\,mg,$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{y}} = \frac{2m}{3}(14\ddot{y} - 5\ddot{x} + 2\ddot{z}) = (-3 + 2 + 1 + \tfrac{10}{3})mg = \tfrac{10}{3}\,mg,$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{z}} = \frac{2m}{3}(8\ddot{z} + 2\ddot{y} - 2\ddot{x}) = (-2 + \tfrac{10}{3})mg = \tfrac{4}{3}\,mg,$$

which give $\ddot{x} = 0$, $\ddot{y} = \frac{1}{3}g$, and $\ddot{z} = \frac{1}{6}g$, and establishes proof sought if we notice that velocity of $3\frac{1}{3}$-pound weight is $\ddot{y} + \ddot{z}$.

Inasmuch as the 9-pound weight always remains at rest if the system starts from rest, the presence of friction in the fixed pulley would not alter the motion of the system as long as no friction be present in the movable pulleys. But if the fixed pulley is frictionless and friction is present in the movable pulleys, then the diminished accelerations of the weights and movable pulleys on the one side of the fixed pulley must, in part, be offset by an upward acceleration of the 9-pound weight on the other side, and in part by the heat of friction generated as long as the friction is not too great to prevent the revolving of the movable pulleys.

352 (Mechanics). Proposed by C. N. SCHMALL, New York City.

A glass rod is balanced partly in and partly out of a cylindrical tumbler, with lower end resting against the vertical wall of the tumbler. If φ and ψ are the maximum and minimum angles, respectively, which the rod can make with the vertical plane, and θ is the angle of friction, show that

$$\theta = \tfrac{1}{2}\tan^{-1}\frac{\sin^3\varphi - \sin^3\psi}{\sin^2\varphi\cos\varphi + \sin^2\psi\cos\psi}.$$

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let $AB = 2a$ be the rod with its lower end at A; G, its middle point; P, the point of support in the edge of the tumbler; CD, the element of the surface of the tumbler in which A lies; and W = the weight of AB. From P draw a perpendicular to CD cutting it in E, and, similarly, draw GK.

The rod in either extreme position is kept in equilibrium by its weight acting vertically downwards, the two reactions R, S, at A and P respectively, and the friction at A in the direction of the element CD and at P in the direction of the rod.

Resolving vertically and horizontally and taking moments about A,

$$W + S\tan\theta\cdot\cos\varphi = S\sin\varphi - R\tan\theta, \tag{1}$$

$$S\cos\varphi + S\tan\theta\cdot\sin\varphi = R, \tag{2}$$

$$W\cdot GK = S\cdot AP, \tag{3}$$

or

$$W\{(a - x)\sin\varphi + b\} = S\cdot x. \tag{4}$$

From the geometry,

$$x = b/\sin\varphi, \tag{5}$$

and this in (4) gives

$$Wa\sin^2\varphi = b. \tag{6}$$